In all cases it can be seen that the increase in conductance due to the presence of the fluid medium is considerable at low contact pressures but the contact spot conductance predominates at higher pressure. This is in agreement with the results of Sanderson [11]. The apparent decrease in gaseous conduction contribution to contact heat transfer as the contact pressure is increased is an anomaly which has also been noted by Ross and Stoute [12] in some of whose tests the fluid conductance was even negative! These results emphasize the difficulties in obtaining accurate information from tests of this kind for reasons mentioned earlier. Also, at high contact pressures both the solid spot conductance and the total conductance are large and the relatively small fluid conductance is determined as a difference between two large quantities. This is likely to be inaccurate.

Figure 5 shows the variation in fluid (air) conductance with fluid pressure for the stainless steel/Nilo combination at four different contact pressures. However, over the small contact pressure range considered (0·755–2·89 MPa), the variation in fluid conductance is small at any given fluid pressure—a fact confirmed by Fig. 2. The results show that the decrease in fluid conductance as the gas pressure is reduced is particularly significant at gas pressures below 100 torr, i.e. when the mean free path for air is greater than about ten times the mean physical gap. Boeschoten and Van der Held [13] observed similar trends.

CONCLUSIONS

(i) The contact conductance improves in the presence of a conducting medium. For all fluids such improvement is significant at low contact pressures; the solid spot conductance predominates at high pressures. When the interface medium is a good conductor such as helium, the improvement is significant over the entire contact pressure range of the tests.

(ii) The results for contacts formed by flat surfaces show agreement with those of previous workers.

(iii) The fluid conduction contribution to heat transfer across a joint at any contact pressure decreases as the fluid pressure is reduced. Such reduction seems to be significant at absolute pressures below 100 torr.

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NOTE ON A PAPER BY KIERKUS

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NOMENCLATURE

g,	acceleration due to gravity;
Gr,	Grashof number $g\beta L^3 \nu^{-2}(T_w - T_\infty)\cos\phi$;
L,	reference length;
p,	pressure;
T,	temperature;
T_w,	wall temperature;
T_∞,	temperature of ambient fluid;
x, y,	Cartesian co-ordinates.

Greek symbols

β,	coefficient of expansion;
θ,	dimensionless temperature $(T - T_\infty)/(T_w - T_\infty)$;
κ,	thermal conductivity;
ν,	kinematic viscosity;
ρ,	density;
σ,	Prandtl number ν/κ;
ϕ,	angle of plate to vertical;
ψ,	stream function.

INTRODUCTION

In this note we re-consider, briefly, the problem of the flow induced when a semi-infinite flat plate, heated to a uniform temperature in excess of the ambient temperature, is inclined at an angle ϕ to the vertical. The Grashof number Gr, defined below, is assumed to be large. The problem has previously attracted the attention of Kierkus [1]. The main feature of the flow when compared with the case $\phi = 0$ is the asymmetry, above and below the plate, due to the normal component of the gravitational field $\mathbf{g}$. The method of solution is to expand the flow quantities in powers of $Gr^{-\frac{1}{4}}$. The structure of the primary boundary layer is identical with that for the symmetric case $\phi = 0$. There is a hydrostatic balance of pressure across the boundary layer which, since heat is continuously absorbed by the fluid as it moves along the plate, results in pressure variations parallel to the plate. Thus for the second-order boundary-layer solution there is a pressure gradient which is favourable above the plate, but adverse below. The leading term of the solution outside the boundary layer given in [1] is incompatible with the primary boundary layer. This we correct by reference to the recent work of Clarke [2], and in addition calculate one further term in the solution outside the boundary layer. This shows that although the anticipated asymmetry is manifest in the second-order boundary-layer solution, the flow described by the two-term outer solution is symmetric to an observer who takes the plate as reference line. We note finally that the solution is formally valid when $((\pi/2)-\phi) = O(1)$ in the limit $Gr \to \infty$. Jones [3] has shown that for an almost horizontal flat plate the self induced pressure gradient is formally comparable with the buoyancy effects.

ANALYSIS

We assume that our fluid is a Boussinesq fluid. We take L as reference length, $[g\beta L(T_w - T_\infty)\cos\phi]^{\frac{1}{2}}$ and $\rho g\beta L(T_w - T_\infty)\cos\phi$ as reference velocity and pressure respectively, where ρ is the density T_w, T_∞ the wall and ambient temperatures and ϕ the angle of inclination of the plate $y = 0$, $x > 0$ to the vertical so that $\mathbf{g} = -g(\cos\phi\mathbf{i} + \sin\phi\mathbf{j})$. Then with $(T - T_\infty) = (T_w - T_\infty)\theta$, and ψ representing the dimensionless stream function, the Navier–Stokes equations may be written as

$$\left.\begin{aligned}
&\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\,\partial y} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} \\
&\qquad = -\frac{\partial p}{\partial x} + \theta + Gr^{-\frac{1}{2}}\frac{\partial}{\partial y}(\nabla^2\psi), \\
&-\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x^2} + \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial x\,\partial y} \\
&\qquad = -\frac{\partial p}{\partial y} + \theta\tan\phi - Gr^{-\frac{1}{2}}\frac{\partial}{\partial x}(\nabla^2\psi), \\
&\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y} \\
&\qquad = \sigma^{-1}Gr^{-\frac{1}{2}}\nabla^2\theta.
\end{aligned}\right\} \quad (1\text{a, b, c})$$

In equations (1) σ is the Prandtl number and $Gr = g\beta L^3\nu^{-2}(T_w - T_\infty)\cos\phi$ the Grashof number, assumed large. The boundary conditions are

$$\left.\begin{aligned}
&\psi = \frac{\partial\psi}{\partial y} = 0, \quad \theta = 1; \quad y = 0, \quad x > 0; \\
&\nabla\psi,\ \theta,\ p \to 0 \quad \text{as} \quad (x^2 + y^2)^{\frac{1}{2}} \to \infty\ (y \neq 0, x > 0).
\end{aligned}\right\} \quad (2)$$

We take advantage of the fact that $Gr \gg 1$ and develop complementary asymptotic solutions in an outer and an inner, or boundary layer, region insisting that the solutions "match" at each stage. Anticipating the form of these we write all variables in the outer region

$$f = Gr^{-\frac{1}{2}}f_2 + Gr^{-\frac{3}{4}}f_3 + \cdots, \quad (3)$$

where $f_i = f_i(x, y)$ and (3) is supposed valid in the limit $Gr \to \infty$, $x, y = O(1)$. For the inner, or boundary layer, region we write

$$\left.\begin{aligned}
\psi &= Gr^{-\frac{1}{4}}(\Psi_1 + Gr^{-\frac{1}{4}}\Psi_2 + \cdots), \\
\theta &= \Theta_1 + Gr^{-\frac{1}{4}}\Theta_2 + \cdots, \\
p &= Gr^{-\frac{1}{4}}P_2 + \cdots.
\end{aligned}\right\} \quad (4)$$

The expansions in (4) are valid for $Gr \to \infty$ with x, $Y(= Gr^{\frac{1}{4}}y)$, $((\pi/2) - \phi) = O(1)$. We note that when, in this limit, $Gr^{\frac{1}{5}}((\pi/2) - \phi) = O(1)$ a different scaling of the variables is required. This case of an almost horizontal flat plate has been discussed by Jones [3]. With the terms $O(1)$ in (3) zero the flow is initiated in the boundary layer, by buoyancy forces acting upon the fluid, as we expect intuitively. If we write

$$\Psi_1 = x^{\frac{3}{4}}f_1(\eta), \quad \Theta_1 = g_1(\eta), \quad \eta = Y/x^{\frac{1}{4}}, \quad (5)$$

then

$$\left.\begin{aligned}
&f_1''' + \tfrac{3}{4}f_1 f_1'' - \tfrac{1}{2}f_1'^2 + g_1 = 0, \quad g_1'' + \tfrac{3}{4}\sigma f_1 g_1' = 0. \\
&\text{with} \\
&f_1(0) = f_1'(0) = 0, \quad g_1(0) = 1; \quad f_1', g_1 \to 0 \quad \text{as} \quad \eta \to \infty.
\end{aligned}\right\} \quad (6)$$

Numerical solutions of (6) are discussed in standard references. We note in particular that for $\sigma = 1$, $f_1(\infty) = 1{\cdot}4792$. We turn next to the terms $O(Gr^{-\frac{1}{2}})$ in the outer solution (3). It can be shown that θ_2, $p_2 \equiv 0$ but the matching requirement shows $\psi_2 \neq 0$. The outer flow is, to this order, irrotational so that ψ_2 satisfies

$$\nabla^2\psi_2 = 0, \quad \psi_2 = \pm x^{\frac{3}{4}}f_1(\infty), \quad y = 0\pm, \quad x > 0. \quad (7)$$

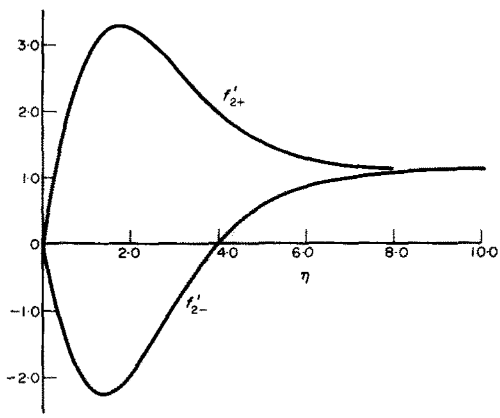

FIG. 1. The second-order velocity functions $f_{2\pm}'(\eta)$, $\sigma = 1$, $\phi = 60°$.

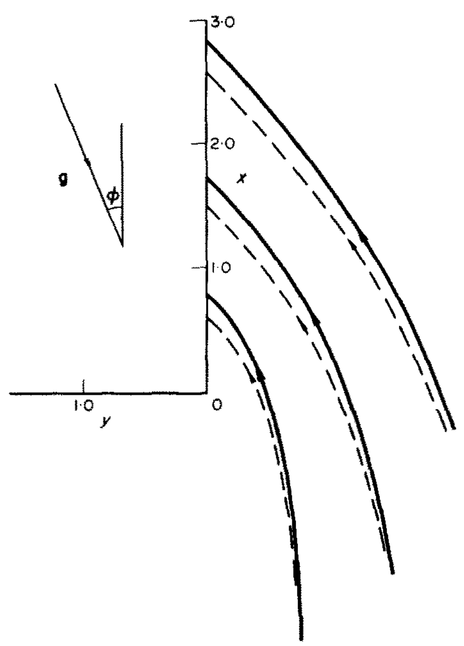

FIG. 2. The streamline pattern ψ = const. in the outer region as calculated from (3), (8) and (13) with $\sigma = 1$, $Gr = 10^4$ and ϕ arbitrary. ---- one term of (3), ——— two terms of (3).

The solution of (7) has been given by Clarke [2] as

$$\psi_2 = (\sqrt{2})f_1(\infty)(x^2+y^2)^{\frac{3}{8}} \sin\left\{\tfrac{3}{4}\tan^{-1}\left(\frac{y}{x}\right)+\frac{\pi}{4}\right\}, \quad (8)$$

which may be compared with [1], equation (24). Kierkus in fact, in his outer solution, attempts to model the effects of a finite plate whilst retaining the solution for a semi-infinite plate in the inner region. His first-order inner and outer solutions are therefore incompatible. The finite-plate problem remains unsolved.

Returning now to the next term of the inner solution we consider (1b) for the pressure distribution. With $P_2 = x^{\frac{1}{2}}h_2(\eta)$ we have for the upper surface

$$h_2' = g_1 \tan\phi, \quad h_2 \to 0 \quad \text{as} \quad \eta \to \infty. \quad (9)$$

This has been given by Kierkus as the leading term of a series associated with the second-order boundary-layer solution. However we note that the remaining terms in his series are meaningless within the context of the incompatibility of his first-order solutions. As Kierkus notes, the pressure distribution in the boundary layer which forms on the lower surface is obtained from (9) by simply changing the sign of ϕ. As for the symmetrical case $\phi = 0$ (see [2]) it can be shown that $\Theta_2 \equiv 0$. The second-order velocity field Ψ_2 within the boundary layer adjusts the "slip" velocity $\partial\psi_2/\partial y|_{y=0} = \frac{3}{4}x^{-\frac{1}{4}}f_1(\infty)$, $(x>0)$ to zero at the plate; its structure is significantly influenced by the self induced pressure field calculated from (9). Thus if we write $\Psi_2 = f_{2\pm}(\eta)$ where $\pm$ refer to the upper and lower sides of the plate respectively, then from (1a)

$$\left.\begin{aligned} f_{2\pm}''' + \tfrac{3}{4}f_1 f_{2\pm}'' - \tfrac{1}{4}f_1' f_{2\pm}' \mp (h_2 - \eta h_2') = 0, \\ f_{2\pm}(0) = f_{2\pm}'(0) = 0, \quad f_{2\pm}'(\infty) = \tfrac{3}{4}f_1(\infty). \end{aligned}\right\} \quad (10)$$

This solution has also been given by Kierkus, as the leading term of an otherwise meaningless series. We show in Fig. 1 the velocity functions $f_{2\pm}'(\eta)$ for $\sigma = 1$, $\phi = 60°$. The asymmetry in the boundary-layer flow brought about by the favourable/adverse pressure gradients on the upper/lower surfaces is clearly demonstrated. To calculate the next term in the outer series (3) we require a matching condition which is derived from the asymptotic form of the solution of (10) as $\eta \to \infty$. Thus $f_{2\pm} \sim \frac{3}{4}f_1(\infty)\eta + c_{2\pm}$ as $\eta \to \infty$ where, for $\sigma = 1$, $\phi = 60°$, $c_{2+} = 6{\cdot}7845$, $c_{2-} = -11{\cdot}2985$. Before proceeding we make the following important observation, namely that $f_2 = \frac{1}{2}(f_{2+} + f_{2-})$ is the solution in the absence of any self-induced pressure field, i.e. $\phi = 0$ as in [2]. Thus if $f_2 \sim \frac{3}{4}f_1(\infty)\eta + c_2$ then

$$c_2 = \tfrac{1}{2}(c_{2+} + c_{2-}). \quad (11)$$

If we now consider the term $O(Gr^{-\frac{1}{2}})$ in (3), it can again be established that the solution is isothermal and irrotational, so that the problem for ψ_3 is

$$\nabla^2\psi_3 = 0, \quad \psi_3 = \pm c_{2\pm}, \quad y = 0\pm, \quad x > 0. \quad (12)$$

The solution of (12) for ψ_3 is

$$\psi_3 = c_{2+} - \frac{1}{2\pi}(c_{2+} + c_{2-})\tan^{-1}\left(\frac{y}{x}\right), \quad (13)$$

from which the purely symmetrical solution is obtained by setting $c_{2\pm} = c_2$. However since the streamlines in the outer flow are given by $\psi = \text{const.}$ we see from (8), (11) and (13) that to an observer outside the boundary layer the symmetrical streamline pattern associated with the case $\phi = 0$ is preserved. Only the 'label' associated with each streamline is changed by the constant amount $\frac{1}{2}(c_{2+} - c_{2-})$. Streamlines calculated from (8), the leading term in (3), are shown in [2]. In Fig. 2 we show the effect upon the streamline pattern of including the second term (13), for a particular value of Gr.

We note finally that since $\Theta_2 \equiv 0$ the heat transfer from the plate is the same as that for a vertical plate up to, but not including, terms of relative order $Gr^{-\frac{1}{2}}$.

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EFFECT OF LIGHTER NONCONDENSABLE GAS ON LAMINAR FILM CONDENSATION OVER A VERTICAL PLATE

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NOMENCLATURE

C_p,	specific heat at constant pressure;
$f(\eta)$,	normalized stream function;
g,	gravity;
h_x,	local heat-transfer coefficient;
k,	thermal conductivity;
M,	molecular weight;
Nu_x,	local Nusselt number;
Pr,	Prandtl number;
Sc,	Schmidt number;
T,	temperature;
u, v,	x and y components of velocity;
W,	mass fraction;
x, y,	parallel and normal direction to the wall.

Greek symbols

λ,	latent heat;
ρ,	density;
β_T,	coefficient of expansion;
μ,	viscosity;
θ,	dimensionless temperature;
ϕ,	dimensionless concentration;
ζ,	ratio of the product of density and viscosity;
η,	similarity variable.

Subscripts and superscripts

1,	condensable vapor;
2,	noncondensable gas;
L,	liquid;
V,	vapor;
i,	interface;
∞,	bulk;

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